

「初中数学都不会」

——Computational Geometrics

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「初中数学都不会」

Outline

- ▶ Basics
 - ▶ Vectors
 - ▶ Distances
 - ▶ Area
 - ▶ Intersections
 - ▶ Transformations
- ▶ Convex hull
- ▶ Extra fun
- ▶ Templates
- ▶ 殴打宇宙智障

Basics

Vectors

- ▶ Computational Geometry in ICPC=
40% Junior high school maths+
30% Senior high school maths+
20% Brainstorming+
10% College maths
- ▶ $\mathbf{u}, \mathbf{v}, \vec{u}, \vec{v}$
- ▶ $\vec{u} + \vec{v}, \vec{u} - \vec{v}, s\vec{u}, \vec{u} \cdot \vec{v}, \vec{u} \times \vec{v}$

Basics

Distances

- ▶ point 2 point
- ▶ point 2 line

Basics

Distances

- ▶ point 2 point
- ▶ point 2 line
- ▶ polygon 2 polygon???

Basics

Area

- ▶ Circle???
- ▶ Triangle?
 - ▶ $ah/2$
 - ▶ Heron's formula
 - ▶ Cross product
- ▶ Polygon???
 - ▶ Slicing

Basics

Intersections

- ▶ point & line
- ▶ point & segment
- ▶ line & line
- ▶ line & segment
- ▶ segment & segment
- ▶ circle & line
- ▶ circle & circle

Basics

Intersections

- ▶ point & line
- ▶ point & segment
- ▶ line & line
- ▶ line & segment
- ▶ segment & segment
- ▶ circle & line
- ▶ circle & circle
- ▶ convex polygon & point

Basics

Intersections

- ▶ point & line
- ▶ point & segment
- ▶ line & line
- ▶ line & segment
- ▶ segment & segment
- ▶ circle & line
- ▶ circle & circle
- ▶ convex polygon & point
 - ▶ randomizing (polyhedron)
 - ▶ ray casting
 - ▶ winding number

Basics

Transformations

▶ Scaling

▶ Rotation

▶
$$\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

▶
$$\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

Convex hull

WTF???

- ▶ Smallest convex polygon (set) that contains a set of points.

Convex hull

Algorithms

- ▶ Brute force: gift wrapping
- ▶ Graham scan

Convex hull

Gift wrapping

- ▶ The left most point must be in the resulting set
- ▶ Starting from that point, select the next point such that all points are **to the right** of the newly formed line.
- ▶ Time complexity: $O(nh)$

Convex hull

Graham scan

- ▶ Find an extreme point and sort the remaining point according to polar angle.
- ▶ Start with a stack with two items, the extreme point and the first point in the sorted list.
- ▶ (P_0 denotes the top element of the stack, P_1 denotes the element under P_0) For each point left in the list (T), pop the stack until $\overrightarrow{P_1P_0}$ and $\overrightarrow{P_0T}$ forms a **left turn**. Then push T onto the stack.
- ▶ Time complexity: $O(n \log n)$ (with sorting)

Convex hull

Example: NEERC 08 A

Extra fun

Example: PetrSU SC 04 C

Arc-arc intersection

PROBLEMS SUBMIT CODE MY SUBMISSIONS STATUS STANDINGS CUSTOM INVOCATION

My Submissions

#	When	Who	Problem	Lang	Verdict	Time	Memory
26373052	2017-04-15 10:11:48	chirs241097	C - Creativity	GNU C++14	Wrong answer on test 4	15 ms	2000 KB
26075584	2017-04-02 13:36:49	chirs241097	C - Creativity	GNU C++14	Accepted	15 ms	2100 KB
26075411	2017-04-02 13:27:09	chirs241097	C - Creativity	GNU C++14	Accepted	15 ms	2100 KB
26073761	2017-04-02 12:01:10	chirs241097	C - Creativity	GNU C++14	Wrong answer on test 8	15 ms	2100 KB
26073622	2017-04-02 11:53:54	chirs241097	C - Creativity	GNU C++14	Wrong answer on test 8	15 ms	2000 KB
26072340	2017-04-02 10:49:31	chirs241097	C - Creativity	GNU C++14	Wrong answer on test 4	15 ms	2100 KB
26072137	2017-04-02 10:38:47	chirs241097	C - Creativity	GNU C++14	Wrong answer on test 4	15 ms	2000 KB
26069587	2017-04-02 08:16:43	chirs241097	C - Creativity	GNU C++14	Wrong answer on test 4	15 ms	2000 KB
26059861	2017-04-01 19:51:49	chirs241097	C - Creativity	GNU C++14	Wrong answer on test 4	15 ms	2000 KB
26058136	2017-04-01 18:40:41	chirs241097	C - Creativity	GNU C++14	Wrong answer on test 4	15 ms	2000 KB
26056970	2017-04-01 17:53:11	chirs241097	C - Creativity	GNU C++14	Wrong answer on test 4	15 ms	2000 KB
26056698	2017-04-01 17:42:13	chirs241097	C - Creativity	GNU C++14	Wrong answer on test 4	15 ms	2000 KB

Extra fun

Example: PetrSU SC 04 C

ID	Who	Contest	When	Problem	Compiler	Result	Time	Memory	
127331	chirs241097	20030827	1:59:19	C_Creativity	GPP 5.1.0	Accepted			Log Src
127330	chirs241097	20030827	1:52:36	C_Creativity	GPP 5.1.0	Presentation error on test #8			Log Src
127329	chirs241097	20030827	1:49:55	C_Creativity	GPP 5.1.0	Wrong answer on test #7			Log Src
127328	chirs241097	20030827	1:46:59	C_Creativity	GPP 5.1.0	Presentation error on test #8			Log Src
127327	chirs241097	20030827	1:46:40	C_Creativity	GPP 5.1.0	Idleness limit on test #1			Log Src
127326	chirs241097	20030827	1:44:53	C_Creativity	GPP 5.1.0	Wrong answer on test #7			Log Src
127325	chirs241097	20030827	1:41:31	C_Creativity	GPP 5.1.0	Wrong answer on test #7			Log Src
127324	chirs241097	20030827	1:40:56	C_Creativity	GPP 5.1.0	Presentation error on test #8			Log Src
127323	chirs241097	20030827	1:40:02	C_Creativity	GPP 5.1.0	Presentation error on test #8			Log Src
127322	chirs241097	20030827	1:37:58	C_Creativity	GPP 5.1.0	Presentation error on test #8			Log Src
127321	chirs241097	20030827	1:35:12	C_Creativity	GPP 5.1.0	Presentation error on test #8			Log Src
127320	chirs241097	20030827	1:32:08	C_Creativity	GPP 5.1.0	Presentation error on test #8			Log Src
127319	chirs241097	20030827	1:31:26	C_Creativity	GPP 5.1.0	Idleness limit on test #1			Log Src
127317	chirs241097	20030827	0:01:47	C_Creativity	GPP 5.1.0	Wrong answer on test #4			Log Src

Extra fun

Example: MultiUniversity Training 20170815 H

Extra fun

Example: Asia GuangzhouRC 14 H

Extra fun

Example: "Angry birds"

One of the few problems of computational geometrics in ICPC that involves college maths.

Extra fun

CG. Combined with other algorithms

Extra fun

Rotating calipers

Solves:

- ▶ Max/min width of convex polygon
- ▶ Max/min distance between two convex polygons
- ▶ Antipodal points/Farthest pairs
- ▶ Union/intersection of convex polygons
- ▶ Min area/perimeter obb
- ▶ etc.

Templates

Critical for Computational Geometrics (maybe)

- ▶ Basic operations
- ▶ Basic intersections
- ▶ Circle-line intersection
- ▶ Circle-triangle intersection
- ▶ Circle-circle intersection
- ▶ 3D vector operations
- ▶ Half-plane intersection
- ▶ Closest/Farthest Pair
- ▶ etc.

殴打宇宙智障

光辉事迹+1

- ▶ 建完图后把图清空然后跑网络流

殴打宇宙智障

光辉事迹+1

- ▶ 建完图后把图清空然后跑网络流
- ▶ 熟悉你的模板

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- ▶ 队伍组成
- ▶ 代码手?

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- ▶ 违反中央八项规定
- ▶ 口技

殴打宇宙智障

后事

- ▶ 21日最后一次讲课
- ▶ 8月31日计科退宿舍
- ▶ 如有不同以宇宙智障为准

殴打宇宙智障

没来得及讲的东西

- ▶ 概率（解方程/dp）、高斯消元
- ▶ 网络流
- ▶ 好像没什么了？
- ▶ Good luck to ya.